



ESHER CHURCH SCHOOL

Christ at the Centre; Life to the Full

Our vision:

To be a safe, happy, loving community where excellent teaching inspires children to learn and explore, care for each other and believe they can make a difference.

PROGRESSION IN CALCULATION GUIDE

Glossary

Array: a regular arrangement of objects

4 x 3 or 3 x 4



Base 10:- cubes used to give a visual representation of 1, 10, 100 and 1000



Bridging: (through 10 or 100)-to use knowledge of number bonds to count on or back to the nearest/best 10 as an efficient strategy to get to the final number eg.. $13 + 8$ as $13 + 7 = 20 + 1 = 21$

Commutativity: that addition or multiplication can be done in any order eg.. 2×3 is the same as 3×2

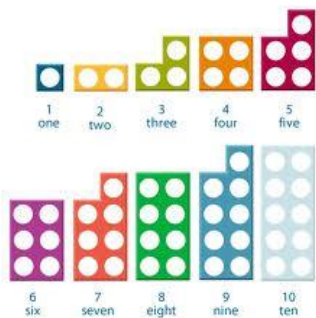
Compensation: to use known number facts to calculate a number that is near to the one known eg.. $27 + 9$ as $27 + 10$ then adjust (compensate) by subtracting 1

Formal written method: a method of calculation with a proven algorithm

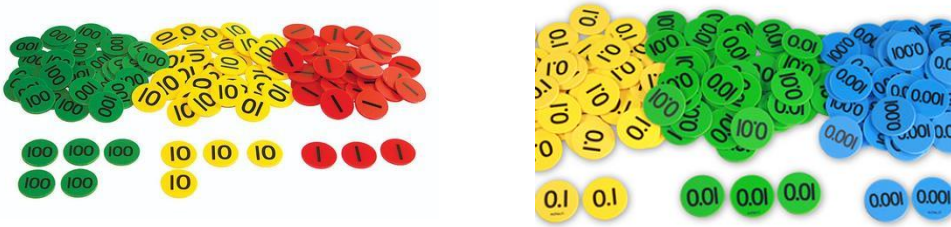
Integers: a complete number either positive or negative, not a fraction or decimal eg.. 3, -3, 103

Inverse: opposite/reverse operations eg.. $4 + 6 = 10$ $10 - 4 = 6$

Numicon: shapes that give a visual representation of numbers from 1-10 and can be combined to make larger numbers



Place value counters: counters that have a numerical value (0.001, 0.01, 0.1, 1, 10, 100 or 1000) marked on them



Whole number: a complete positive number e.g. 3, 41, 1007; not negative, fraction or decimal

For more definitions visit:

<http://www.amathsdictionaryforkids.com/dictionary.html>

This guide has calculation methods for all 4 operations and is designed to be used in a linear progression. The year groups attached *are guidelines* in line with the National Curriculum however children should use the strategy appropriate to their development as assessed by their class teacher. The children will be encouraged to draw upon strategies they have learned in previous years as this helps to consolidate their use and understanding of the methods.

Throughout their lessons, children will be given opportunities to apply these calculation methods in a range of problem solving contexts. This is paramount in helping the children become secure with each method. By secure, we mean they can understand and explain each method clearly and in particular have confident understanding of the place value involved with the methods.

Secure knowledge of number facts and in particular place value are *key* to children's success in mastering written methods and therefore is a **key priority**.

Moving on to the next calculation method (particularly from another year group) will be at the discretion of the class teacher as they have the greatest knowledge of the children's understanding. Despite formal written methods being seemingly easy to learn, they are challenging to understand and apply if a secure knowledge and understanding of place value is not in place. Children should not move onto the next stage if they are not ready, not confident and/or show a lack of understanding of the method.

Children should be encouraged to approximate their answers before calculating and use their understanding of number and place value to check their answers.

By the end of year 6, children should feel secure range of mental and written calculation methods. Selection of method will depend upon the numbers involved. Children will be taught how and encouraged to select the most appropriate method throughout each year group as they learn new methods.

Addition

Counting altogether from 0 (combining) and then counting on *initially verbally and with concrete resources and progressing to writing numerals*:

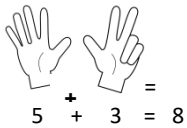
Using objects:



$$4 + 3 = 7$$

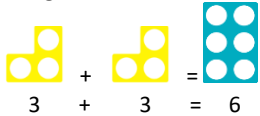
$$7 + 3 = 10$$

Using fingers:



$$5 + 3 = 8$$

Using Numicon:



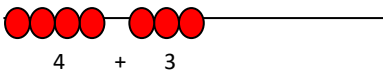
$$3 + 3 = 6$$

Drawing pictures:



$$3 + 2 = 5$$

Counting on a bead bar/string:



$$4 + 3$$

Counting on a number line in 1s with fingers:



$$3 + 2 = 5$$

Subtraction

Removing, covering, crossing off and counting back *initially verbally and with concrete resources and progressing to writing numerals*:

Using objects (taking them away):



$$7 - 2 = 5$$

Using fingers (counting back, folding fingers down):



$$5 - 2 = 3$$

Using Numicon (overlying or covering up):

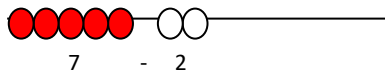


$$6 - 2 = 4$$

Drawing pictures (crossing off):



Counting on a bead bar/string:



$$7 - 2$$

Counting back on a number line in 1s with fingers:



$$7 - 2 = 5$$

Multiplication

Counting in groups of 2, 5 and 10 using concrete objects, singing songs using concrete objects:

Using objects:



2 4 6 8 10 12 14 16

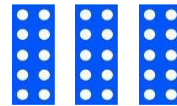


5

10

15

Using Numicon (repeated addition verbally):

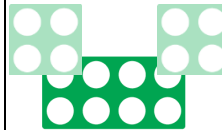


10

20

30

Doubling (overlying Numicon):



Division

Halving using vocabulary of sharing:

Using objects (sorting into 2 groups):



Half of 8 cubes is 4 cubes

Using Numicon:



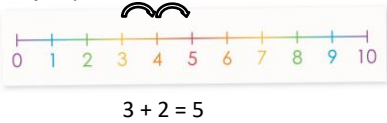
Using real life examples e.g. pizza:



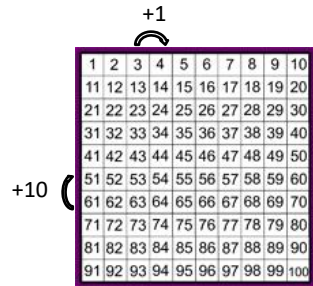
Addition

Counting on continuing to use the objects and resources introduced in Reception *and progressing to*:

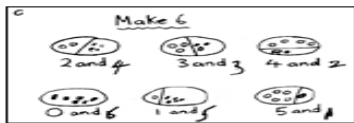
Counting on a number line in 1s by drawing the jumps:



Counting on a hundred square in 1s and 10s (place value understanding of jumping down rows):



Drawing pictures (particularly when beginning to solve problems/investigate numbers):



Using balances to write calculations e.g. 3 + 7 = 10:

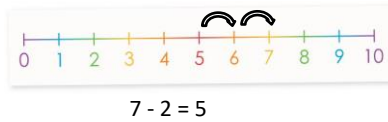


Using known facts:
e.g. 3 + 5 = 8 so
13 + 5 = 18 and
23 + 5 = 8 e.t.c.

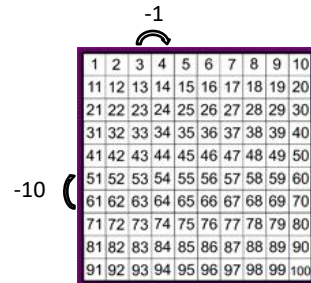
Subtraction

Removing, covering, crossing off and counting back continuing to use the objects and resources introduced in Reception *and progressing to*:

Counting back on a number line in 1s by drawing the jumps:



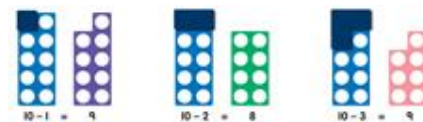
Counting back on a hundred square in 1s and 10s (place value understanding of jumping up rows):



Drawing pictures (particularly when beginning to solve problems/investigate numbers):



Overlaying Numicon to write calculations:

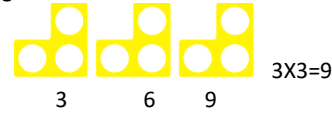


Using known facts:
e.g. 7 - 3 = 4 so
17 - 3 = 14 and
27 - 3 = 24 e.t.c.

Multiplication

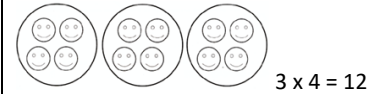
Counting in groups continuing to use the objects and resources introduced in Reception *and progressing to* using repeated addition:

Using Numicon:

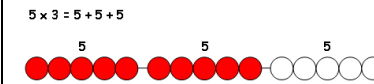


3 times 3 is 3 + 3 + 3 = 9 or 3 lots of 3 or 3 x 3

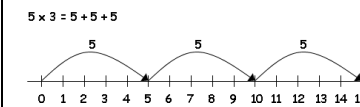
Using objects and pictures:



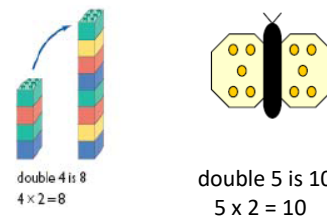
On a bead bar/string:



On a number line (repeated addition):



Doubling using objects and pictures:



Division

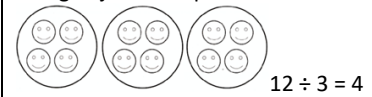
Splitting into groups continuing to use the objects and resources introduced in Reception *and progressing to* using repeated subtraction:

Using Numicon:

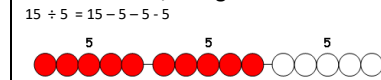


12 divided by 3 is 12 - 3 - 3 - 3 - 3 = 0 or 12 ÷ 3

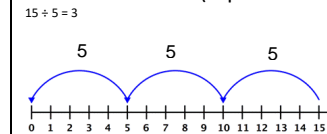
Using objects and pictures:



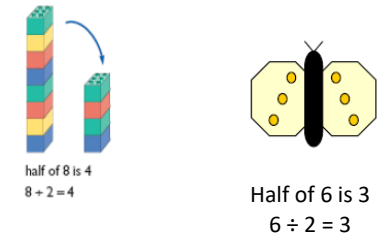
On a bead bar/string:



On a number line (repeated subtraction):



Halving (sharing) using objects and pictures:

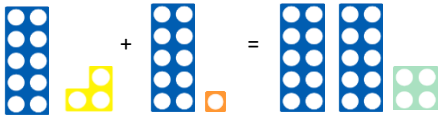


Addition

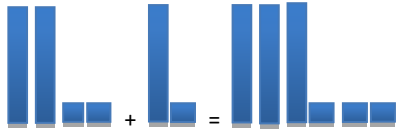
Counting on continuing to use a number line and hundred square and *progressing to*:

Using partitioning (adding tens and ones/units separately):

With Numicon:



With Base 10:

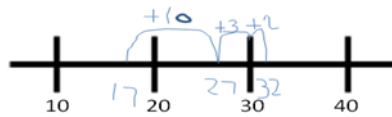


Using a bead string for bridging:
 $8+5 (8+2+3)=10$



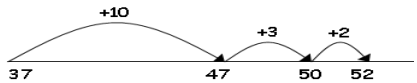
Using a landmarked number line for counting on in 10s, counting on units in one jump and bridging (taught in graduated steps):

$17 + 15$



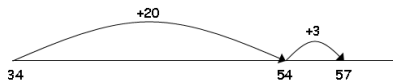
Using an empty number line for counting on in 10s and bridging:

$37 + 15 = 52$



Using an empty number line for counting on in multiples of 10 (and bridging):

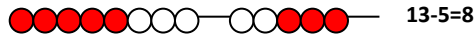
$34 + 23 = 57$



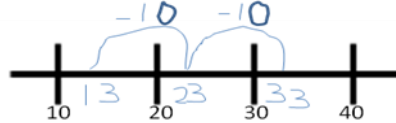
Subtraction

Counting back continuing to use a number line and hundred square and *progressing to*:

Using a bead string for bridging:



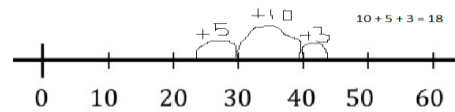
Using a landmarked number line for subtracting 10s:
 $33 - 20 =$



Counting up using bridging and adding 10s:

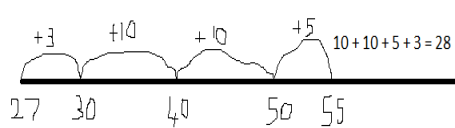
Using a landmarked number line:

$43 - 25 =$



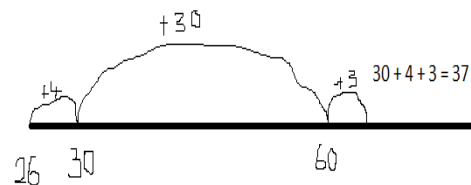
Using an empty number line:

$55 - 27$



Counting up using bridging and multiples of 10:

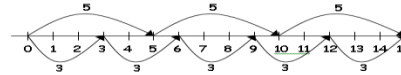
$63 - 26$



Multiplication

Continuing to use repeated addition on a number line and counting in groups using methods taught in Year One and *progressing to* using an array. Children can also recognise the commutative relationship between multiplication facts:

Using a number line:

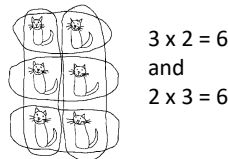


Using objects:



$4 \times 3 = 12$
and
 $3 \times 4 = 12$

Drawing pictures:



$3 \times 2 = 6$
and
 $2 \times 3 = 6$

Using real life examples:



$3 \times 4 = 12$
and
 $4 \times 3 = 12$

Drawing an array:

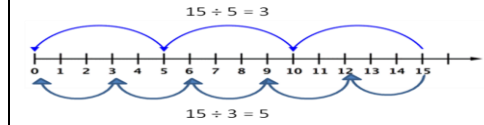


$4 \times 3 = 12$ $3 \times 4 = 12$

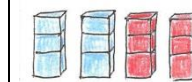
Division

Continuing to use repeated subtraction on a number line and splitting into groups using methods taught in Year One and *progressing to* using an array.

Using a number line:

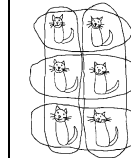


Using objects:



$12 \div 4 = 3$
and
 $12 \div 3 = 4$

Drawing pictures:



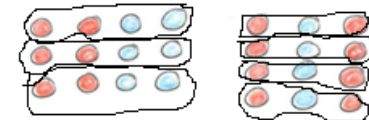
$6 \div 2 = 3$
and
 $6 \div 3 = 2$

Using real life examples:



$12 \div 3 = 4$
and
 $12 \div 4 = 3$

Drawing an array:

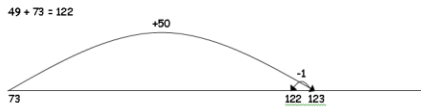


$12 \div 4 = 3$ $12 \div 3 = 4$

Addition

Counting on continuing to use the empty number line with multiples of 10 and *progressing to:*

Using the number line to compensate:



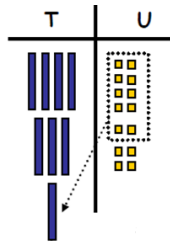
Using expanded addition (partitioning):
32 + 26

T	U
30	+ 2
20	+ 6
50	+ 8

Using expanded column addition with Base 10 to support understanding of exchanging 'carrying' initially:

48 + 36

T	U
40	+ 8
30	+ 6
80	+ 4
10	



Becoming secure in using these methods with three digit numbers:

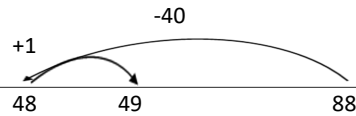
466 + 358

H	T	U
400	60	6
+ 300	50	8
<hr/>		
700	110	14

Subtraction

Counting up continuing to use the empty number line with multiples of 10 and *progressing to:*

Using the number line to compensate:
88 - 39



Using expanded subtraction:

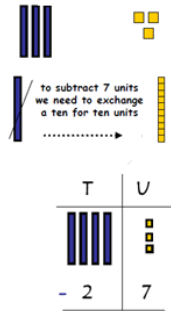
89 - 57

80	+ 9
- 50	+ 7
<hr/>	
30	+ 2 = 32

Using expanded column subtraction (decomposition) with Base 10 to support understanding of 'exchanging' initially:

43 - 27 = 16

30	40	+ 10	+ 3
- 20	+ 7		
<hr/>			
10	+ 6		



Becoming secure in using these methods with three digit numbers:

354 - 146

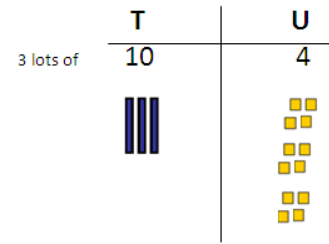
H	T	U
300	50	4
- 100	40	6
<hr/>		
200	+ 0	+ 8

Multiplication

Continuing to use arrays and multiplication facts that they know and *progressing to:*

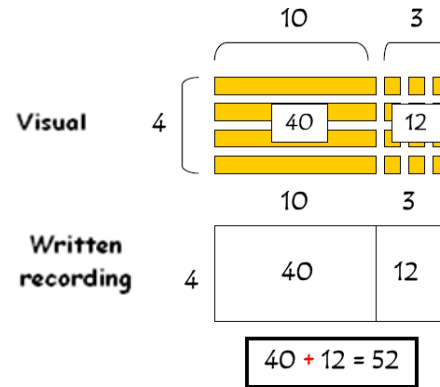
Using Base 10 practically:

3 x 14

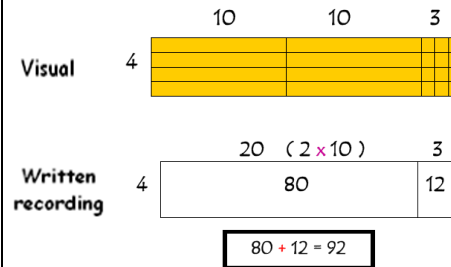


Using the grid method with Base 10:

4 x 13



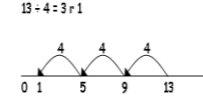
and... **4 x 23**



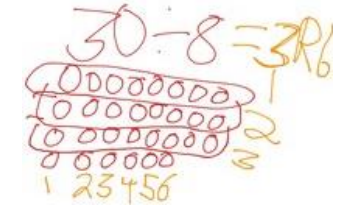
Division

Continuing to use repeated subtraction on a number line and arrays and *progressing to:*

Finding remainders on a number line:

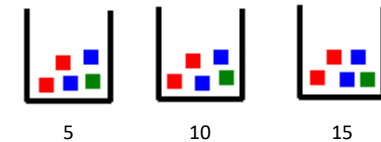


Finding remainders using an array:



Grouping into cups (practically initially to support understanding):

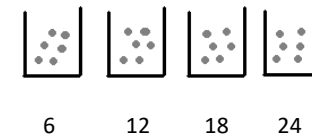
15 ÷ 3



5 cubes in each cup means **15 ÷ 3 = 5**

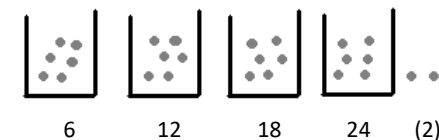
Grouping into cups drawn by children:

24 ÷ 4 = 6 (dots in each cup)



Grouping into cups to find remainders:

26 ÷ 4 = 6 r 2 (6 dots in each cup with 2 left over)



Addition

Continuing to select the number line for appropriate calculations and *progressing to*:

Using formal column addition with exchanging 'carrying':

$$\begin{array}{r} 48 \\ + 36 \\ \hline 84 \\ \hline \end{array}$$

Becoming secure in this method to add three or more numbers:

$$\begin{array}{r} 37 \\ + 28 \\ + 19 \\ \hline 84 \\ \hline \end{array}$$

Becoming secure in using this method with three and four digit whole numbers:

$$\begin{array}{r} 243 \\ + 597 \\ \hline 840 \\ \hline \end{array}$$

Using expanded column addition to add numbers with up to two decimal places with place value counters to support understanding of exchanging 'carrying' initially:

U	Tths	Hths
2	0.2	0.08
1	0.1	0.03
3	0.3	0.11

Using formal column addition to add numbers with up to two decimal places:

$$\begin{array}{r} 12.79 \\ + 5.97 \\ \hline 18.76 \\ \hline \end{array}$$

Subtraction

Continuing to select the number line for appropriate calculations and *progressing to*:

Using formal column subtraction (decomposition) with exchanging:

$$\begin{array}{r} 34 \\ - 27 \\ \hline 7 \\ \hline \end{array}$$

Becoming secure in using this method with three and four digit whole numbers including those involving a digit value of 0:

$$\begin{array}{r} 8948 \\ - 263 \\ \hline 8685 \\ \hline \end{array}$$

$$\begin{array}{r} 31 \\ - 1385 \\ \hline 1071 \\ \hline \end{array}$$

$$\begin{array}{r} 819171 \\ - 4789 \\ \hline 4291 \\ \hline \end{array}$$

Using expanded column subtraction to subtract numbers with up to two decimal places with place value counters to support understanding of exchanging initially:

U	Tths	Hths
2	0.2	0.04
1	0.1	0.07
1	0.0	0.07

Using formal column subtraction to subtract numbers with up to three decimal places:

$$\begin{array}{r} 171 \\ - 1287 \\ \hline 538 \\ \hline \end{array}$$

Multiplication

Continuing to use the grid method and multiplication facts they know and *progressing to*:

Using place value counters to support understanding:

16 x 3

x	10	6	
3	30	18	= 48

Using the grid method to multiply two-digit and three-digit numbers by a single digit:

x	20	8
7	140	56

$$140 + 56 = 196$$

Using the expanded formal column method to multiply two-digit and three-digit numbers by a single digit:

$$38 \times 7 =$$

$$\begin{array}{r} 30 + 8 \\ \times 7 \\ \hline 56 \\ 210 \\ \hline 266 \end{array}$$

(7 x 8 = 56)
(7 x 30 = 210)

Using the formal column method to multiply two-digit and three-digit numbers by a single digit:

$$\begin{array}{r} 39 \\ \times 8 \\ \hline 312 \\ \hline \end{array}$$

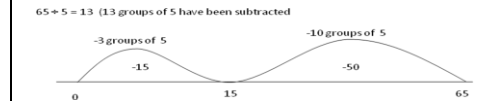
Division

Continuing to use the cup method and *progressing to*:

Using cups to group in larger amounts 'chunks' (using place value counters):

Using cups to group in larger amounts 'chunks' (using multiplication facts):

Using a number line to jump in larger groups 'chunks' (using multiplication facts):



Using a vertical 'chunking' method to divide two and three digit numbers by a single digit (using multiplication facts):

$$91 \div 7 = 13$$

$$\begin{array}{r} 7 \overline{)91} \\ - 70 \quad (10 \times 7) \\ \hline 21 \\ - 21 \quad (3 \times 7) \\ \hline 0 \end{array}$$

Fact box:

$$1 \times 7 = 7$$

$$2 \times 7 = 14$$

$$10 \times 7 = 70$$

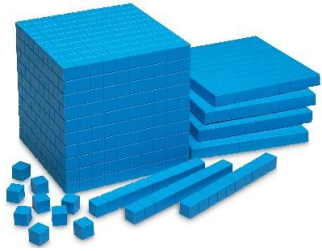
$$5 \times 7 = 35$$

$$10 + 3 = 13$$

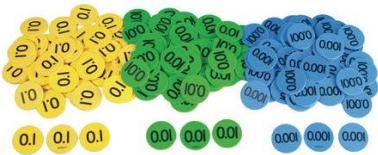
Addition

Continuing to use the formal column addition method for calculations involving whole numbers greater than 4 digits and for numbers with up to 3 decimal places:

Using Base 10 to support understanding:



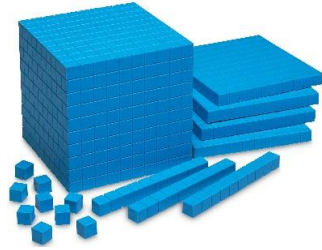
Using decimal place value counters to support understanding:



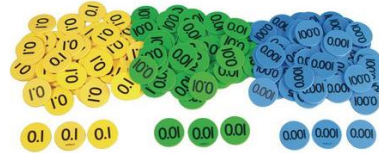
Subtraction

Continuing to use the formal column subtraction method for calculations involving whole numbers greater than 4 digits and for numbers with up to 3 decimal places:

Using Base 10 to support understanding:



Using decimal place value counters to support understanding:



Multiplication

Continuing to use the grid method and formal written method as well as the multiplication facts up to 12 x 12 and *progressing to*:

Using the grid method to multiply two-digit and three-digit numbers by a two-digit number (using place value counters if necessary):

x	30	4	
10	300	40	340
6	180	24	204
340 + 204 = 544			

Using an expanded formal column method to multiply two-digit and three-digit numbers by a two-digit number:

$$\begin{array}{r} 56 \\ \times 14 \\ \hline 24 \\ 200 \\ 60 \\ \hline 500 \\ 784 \end{array}$$

$4 \times 6 = 24$ $4 \times 50 = 200$ $10 \times 6 = 60$ $10 \times 50 = 500$
--

Using the formal column method (long multiplication) to multiply two-digit and three-digit numbers by a two-digit number:

$$\begin{array}{r} \text{TU} \\ 56 \\ \times 14 \\ \hline 224 \\ 60 \\ \hline 784 \end{array}$$

<p>Multiply each digit in 56 by 4 units (ones) Place a 0 in the units column to show you are now multiplying by a tens number TIP: Some children find it helpful to record the tens and units in differing colours. Multiply each digit in 56 by 1 ten Add using column addition and record.</p>

Division

Continuing to use a vertical 'chunking' method to divide a two and three digit number by a single digit and *progressing to*:

Using the formal short division method for dividing a two, three and four digit number by a single digit:

$$81 \div 3 = 27$$

$$\begin{array}{r} 27 \\ 3 \overline{) 81} \end{array}$$

Using a vertical 'chunking' method to divide two and three digit numbers by a single digit (using multiplication facts):

$$406 \div 29 = 14$$

$$\begin{array}{r} 14 \\ 29 \overline{) 406} \\ \underline{- 290} \\ 116 \\ \underline{- 116} \\ 0 \end{array}$$

<p>Fact box: $1 \times 29 = 29$ $2 \times 29 = 58$ $10 \times 29 = 290$ $5 \times 29 = 145$</p>
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Addition

Children should be confident in selecting the most appropriate method of calculation (mental or written) from those included in this policy and particularly including:

- Using number bonds to 1, 10, 20, 100 and 100
- Using partitioning
- Using a number line both mentally and in written form
- Using bridging through a number
- Using compensating
- Using the formal column method

Subtraction




Children should be confident in selecting the most appropriate method of calculation (mental or written) from those included in this policy and particularly including:

- Using number bonds to 1, 10, 20, 100 and 100
- Using partitioning
- Using a number line both mentally and in written form
- Using bridging through a number
- Using compensating
- Using the formal column method

Multiplication

Continuing to use the grid method and formal method to multiply up to 4-digit whole numbers by 2-digit numbers and *progressing to*:

Using the grid method with place value counters initially to multiply numbers with up to two decimal places by a single digit:

	Units/Ones	Tenths	Hundredths
x	1	3	2
4			
	4 Units/Ones	12 Tenths	8 Hundredths

4 Units + 12 Tenths + 8 Hundredths = 5.28

Using the formal column method to multiply numbers up to two decimal places by a single digit:

$$\begin{array}{r} 1.32 \\ \times 4 \\ \hline 5.28 \\ 1 \end{array}$$

Division

Continuing to use the short division method for dividing by single digits and *progressing to*:

Finding a decimal remainder using the formal short division method (dividing by both a single and two-digit number):

$116 \div 8 = 14.5$

$$\begin{array}{r} 014.5 \\ 8 \overline{)116.0} \end{array}$$

Using the formal long division method:

$748 \div 51 =$

$\begin{array}{r} 14 \\ 51 \overline{)748} \\ -51 \\ \hline 238 \\ -204 \\ \hline 34 \end{array}$	<p>$74 \div 51 = 1$ remainder 23</p> <p>Subtract (1 group of) 51 from 74</p> <p>Write 1 above the 4 in 748</p> <p>Write the remainder underneath</p> <p>Bring the 8 down from 748</p> <p>Write it next to 23</p> <p>$238 \div 51 = 4$ remainder 34</p> <p>(Use approximation for this e.g. $4 \times 50 = 200$ so $4 \times 51 = 204$)</p> <p>Subtract (4 groups of 51) 204 from 238</p> <p>Write 4 above the 8 in 748</p> <p>Write the remainder underneath</p>
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Finding a decimal remainder using the long division method:

$748 \div 51$

$$\begin{array}{r} 14.66 \\ 51 \overline{)748.00} \\ - 51 \\ \hline 238 \\ - 204 \\ \hline 34.0 \\ - 30.6 \\ \hline 3.40 \\ - 3.06 \\ \hline 0.34 \end{array}$$